Application of Temporal Logic to Bitemporal Database containing Medical Data

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Time

Time is a structure $\mathfrak{I} = (T, <)$

where T - a non empty set of points (dates),

$<$ - a binary relation on T, i.e., $< \subseteq T \times T$

Bitemporal Database

Bitemporal Database stores the valid time and the transaction time associated with each piece of data.

Valid time of data - dates when the fact represented by data is true in the modelled reality

Transaction time of data - dates when data is held in the database

The model of bitemporal database

$\mathbf{DB} = (T, <_t, \underline{T}, <_{\underline{t}}, D, R_1, \ldots, R_n, (M_i)_{i \in \{1,\ldots,n\}})$

T - a set of dates to describe the valid time

$\underline{T}$ - a set of dates to describe the transaction time

$<_t \subseteq T \times T; <_{\underline{t}} \subseteq \underline{T} \times \underline{T};$

D - data domain

R_i - a finite relation on D; $R_i \subseteq \underbrace{D \times \ldots \times D}_{k_i\text{-times}}$

M_i - a function marking data with dates, i.e., $M_i : R_i \rightarrow 2^{(2^T \times 2^{\underline{T}})}$

Example (1)

$T = \underline{T}$ - the set of dates, granularity is one day

$<_t$, $\leq_t$ - natural order in the set of dates

D - the set of identifiers, surnames and the names of medicines

PATIENTS(*pid*, *surname*) =
{(1,Kowalski),(2,Kozłowski),(3,Nowak)} $\subseteq D^2$

TREATMENT(*pid*, *medicine*) =
{(1,A),(2,B),(3,A)} $\subseteq D^2$

Example (2)

M_P[(1,Kowalski)] = {([10-26V],[1-3V]),([13-26V],[4V-now])}

M_P[(2,Kozłowski)] = {([15-17V],[4V-now])}

M_P[(3,Nowak)] = {([1-3VI],[27V-1VI])}

M_T[(1,A)] = {([13-26V],[10V-now])}

M_T[(2,B)] = {([18-20V],[10-16V]),([20-27V],[17V-now])}

M_T[(3,A)] = {([1-2VI],[27V-1VI])}

VALID TIME:

- dates of stay of a patient in the hospital (relation PATIENTS)
- dates of treatment of a patient (relation TREATMENT)

TRANSACTION TIME:

- dates when information (record) about stay of a patient in the hospital is held in database (relation PATIENTS)
- dates when information (record) about treatment of a patient is held in database (relation TREATMENT)

The temporal logic language

Alphabet

- domain variables: $x_1, x_2, \ldots$;
- domain constants: $c_1, c_2, \ldots$;
- time variables: $t_1, t_2, \ldots$;
- time constants: $e_1, e_2, \ldots$;
- predicate symbols: $P_1, P_2, \ldots, P_k$;
- equality symbol: $=$;
- logical connectives: $\neg$, $\wedge$;
- existential quantifier: $\exists$;
- temporal connectives: **U**, **S** **<u>U</u>**, **<u>S</u>**;
- punctuation symbols: *),(*.
- additional predicate symbols: *date*, <u>*date*</u>;

Syntax

Term: a constant or a variable

Atomic formula:

- $P_i(a_1, \ldots, a_n)$, where P_i - a predicate symbol with the arity n,
  $a_1, \ldots, a_n$ - domain terms
- $a_i = a_j$, where a_i , a_j - terms of the same sort, i.e., domain or time terms
- $date(a_i)$, $\underline{date}(a_i)$, where a_i - time term

Well Formed Formula (wff):

- atomic formula is wff
- **if** φ, ψ are wff
  then $\neg\varphi$, $\varphi \wedge \psi$, $\exists x\, \varphi$, $\varphi\ \mathbf{U}\ \psi$, $\varphi\ \mathbf{S}\ \psi$, $\varphi\ \underline{\mathbf{U}}\ \psi$, $\varphi\ \underline{\mathbf{S}}\ \psi$
  are wff (x - domain variable)

Interpretation (I)

$I(P_i) = R_i$ (predicate symbols as relations of DB)

$I(c_i) \in D$ (domain constants as elements of data domain)
$I(e_i) \in T \cup \underline{T}$ (time constants as dates)

Assignment θ:
domain terms are associated to the elements of D
time terms are associated to the elements of $T \cup \underline{T}$

We assume that $\theta(a) = I(a)$, where *a* is constant.

Semantics

(1) $DB, \theta, t, \underline{t} \models P_i(x_1, \ldots, x_n)$ **iff**

there exists $(A,B) \in M_i[(\theta(x_1), \ldots, \theta(x_n)]$: $t \in A$ **and** $\underline{t} \in B$

(2) $DB, \theta, t, \underline{t} \models a_i = a_j$ **iff** $\theta(a_i) = \theta(a_j)$

(3) $DB, \theta, t, \underline{t} \models \neg \varphi$ **iff** not $DB, \theta, t, \underline{t} \models \varphi$

(4) $DB, \theta, t, \underline{t} \models \varphi \wedge \psi$ **iff**

$DB, \theta, t, \underline{t} \models \varphi$ **and** $DB, \theta, t, \underline{t} \models \psi$

(5) $DB, \theta, t, \underline{t} \models \exists x\, \varphi$ **iff**

there exists an assignment θ^* (that associates all the variables except, possibly, the variable *x* to the same values as θ)

such that $DB, \theta^*, t, \underline{t} \models \varphi$

(6) $DB, \theta, t, \underline{t} \models date(a_i)$ **iff** $\theta(a_i) = t$

(7) $DB, \theta, t, \underline{t} \models \underline{date}(a_i)$ **iff** $\theta(a_i) = \underline{t}$

(8) $DB, \theta, t, \underline{t} \models \psi \; \mathbf{U} \; \varphi$ **iff**

there exists $t_1 \in T$:

$t < t_1$ **and** $DB, \theta, t_1, \underline{t} \models \varphi$

and for every $t_2 \in T$: **whenever** $t < t_2 < t_1$ **then** $DB, \theta, t_2, \underline{t} \models \psi$

(9) $DB, \theta, t, \underline{t} \models \psi \; \mathbf{S} \; \varphi$ **iff**

there exists $t_1 \in T$:

$t_1 < t$ **and** $DB, \theta, t_1, \underline{t} \models \varphi$

and for every $t_2 \in T$: **whenever** $t_1 < t_2 < t$ **then** $DB, \theta, t_2, \underline{t} \models \psi$

(10) $DB, \theta, t, \underline{t} \models \psi \; \underline{\mathbf{U}} \; \varphi$ **iff**

there exists $\underline{t}_1 \in \underline{T}$:

$\underline{t} < \underline{t}_1$ **and** $DB, \theta, t, \underline{t}_1 \models \varphi$

and for every $\underline{t}_2 \in \underline{T}$: **whenever** $\underline{t} < \underline{t}_2 < \underline{t}_1$ **then** $DB, \theta, t, \underline{t}_2 \models \psi$

(11) $DB, \theta, t, \underline{t} \models \psi \; \underline{\mathbf{S}} \; \varphi$ **iff**

there exists $\underline{t}_1 \in \underline{T}$:

$\underline{t}_1 < \underline{t}$ **and** $DB, \theta, t, \underline{t}_1 \models \varphi$

and for every $\underline{t}_2 \in \underline{T}$: **whenever** $\underline{t}_1 < \underline{t}_2 < \underline{t}$ **then** $DB, \theta, t, \underline{t}_2 \models \psi$

Query:

A formula of temporal logic φ with at least one free variable.

The answer of a query φ:

$Q(DB,\varphi) = \{(\theta(x_1), ..., \theta(x_n)) : DB, \theta, t, \underline{t} \models \varphi\}$,

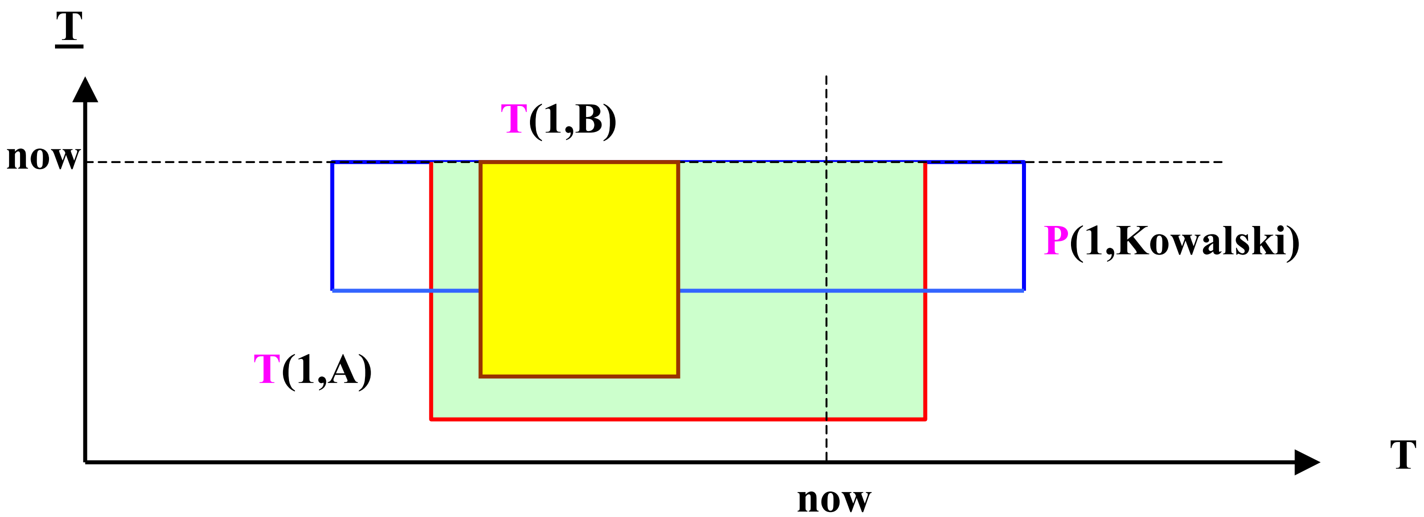
where $x_1, ..., x_n$ are the only free variables of formula φ

Example of queries (1)

$\varphi = [T(1,x)\ \mathbf{S}\ P(1,Kowalski)] \wedge date(now) \wedge \underline{date}(now)$

$Q(DB,\varphi) = \{\theta(x): DB,\theta,now,now \models T(1,x)\ \mathbf{S}\ P(1,Kowalski)\}$

there exists $t_1 \in$ T:
$t_1 < now$ **and** DB, θ, t_1, now $\models$ P(1,Kowalski)
and for every $t_2 \in$ T: **whenever** $t_1 < t_2 < now$ **then** DB, θ, t_2, now $\models$ T(1,x)



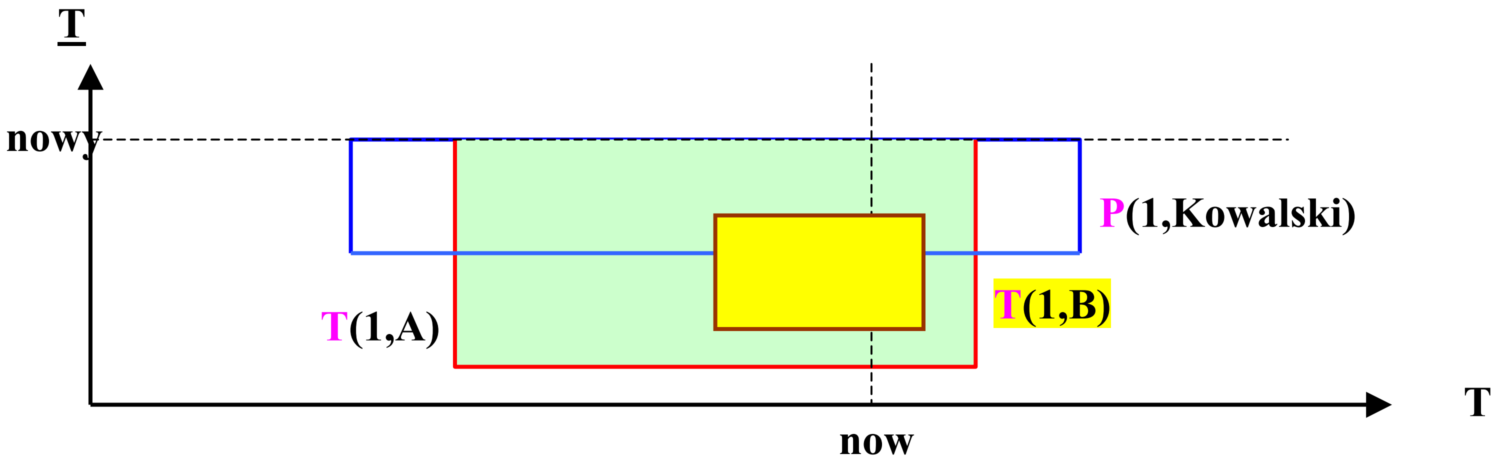
We search for the names of medicine that Kowalski has been treated with at least until yesterday.

Example of queries (2)

$\varphi = [T(1,x) \; \underline{\mathbf{S}} \; P(1,Kowalski)] \wedge date(now) \wedge \underline{date}(now)$

$Q(DB,\varphi) = \{\theta(x): DB,\theta,now,now \models T(1,x) \; \underline{\mathbf{S}} \; P(1,Kowalski)\}$

there exists $\underline{t}_1 \in \underline{T}$:
$\underline{t}_1 <$ now **and** DB, θ, now, $\underline{t}_1 \models P(1,Kowalski)$
and for every $\underline{t}_2 \in \underline{T}$: **whenever** $\underline{t}_1 < \underline{t}_2 <$ now **then** DB, θ, now, $\underline{t}_2 \models T(1,x)$



We search for the records (still present in the database) of current treatment of Kowalski that have been inserted since the record of Kowalski's stay in the hospital is held in the database.

Other temporal connectives

$\mathbf{F}\varphi := \top\ \mathbf{U}\ \varphi$

$\mathbf{P}\varphi := \top\ \mathbf{S}\ \varphi$

$\mathbf{G}\varphi := \neg\mathbf{F}\neg\varphi$

$\mathbf{H}\varphi := \neg\mathbf{P}\neg\varphi$

$\mathbf{X}\varphi := \bot\ \mathbf{U}\ \varphi$

$\mathbf{X}^{-}\varphi := \bot\ \mathbf{S}\ \varphi$

Other temporal connectives - semantics (1)

(1) $DB, \theta, t, \underline{t} \models \mathbf{F}\varphi$ **iff**

there exists $t_1 \in T$: $t < t_1$ **and** $DB, \theta, t_1, \underline{t} \models \varphi$

(2) $DB, \theta, t, \underline{t} \models \mathbf{P}\varphi$ **iff**

there exists $t_1 \in T$: $t_1 < t$ **and** $DB, \theta, t_1, \underline{t} \models \varphi$

(3) $DB, \theta, t, \underline{t} \models \mathbf{G}\varphi$ **iff**

for every $t_1 \in T$: **whenever** $t < t_1$ **then** $DB, \theta, t_1, \underline{t} \models \varphi$

(4) $DB, \theta, t, \underline{t} \models \mathbf{H}\varphi$ **iff**

for every $t_1 \in T$: **whenever** $t_1 < t$ **then** $DB, \theta, t_1, \underline{t} \models \varphi$

(5) $DB, \theta, t, \underline{t} \models \mathbf{X}\varphi$ **iff**

there exists $t_1 \in T$: $t_1 = t+1$ **and** $DB, \theta, t_1, \underline{t} \models \varphi$

(6) $DB, \theta, t, \underline{t} \models \mathbf{X}^{-}\varphi$ **iff**

there exists $t_1 \in T$: $t_1+1 = t$ **and** $DB, \theta, t_1, \underline{t} \models \varphi$

Other temporal connectives - semantics (2)

(1) $DB, \theta, t, \underline{t} \models \underline{\mathbf{F}}\varphi$ **iff**

there exists $\underline{t}_1 \in \underline{T}$: $\underline{t} < \underline{t}_1$ **and** $DB, \theta, t, \underline{t}_1 \models \varphi$

(2) $DB, \theta, t, \underline{t} \models \underline{\mathbf{P}}\varphi$ **iff**

there exists $t_1 \in \underline{T}$: $\underline{t}_1 < \underline{t}$ **and** $DB, \theta, t, \underline{t}_1 \models \varphi$

(3) $DB, \theta, t, \underline{t} \models \underline{\mathbf{G}}\varphi$ **iff**

for every $\underline{t}_1 \in \underline{T}$: **whenever** $\underline{t} < \underline{t}_1$ **then** $DB, \theta, t, \underline{t}_1 \models \varphi$

(4) $DB, \theta, t, \underline{t} \models \underline{\mathbf{H}}\varphi$ **iff**

for every $\underline{t}_1 \in \underline{T}$: **whenever** $\underline{t}_1 < \underline{t}$ **then** $DB, \theta, t, \underline{t}_1 \models \varphi$

(5) $DB, \theta, t, \underline{t} \models \underline{\mathbf{X}}\varphi$ **iff**

there exists $\underline{t}_1 \in \underline{T}$: $\underline{t}_1 = \underline{t}+1$ **and** $DB, \theta, t, \underline{t}_1 \models \varphi$

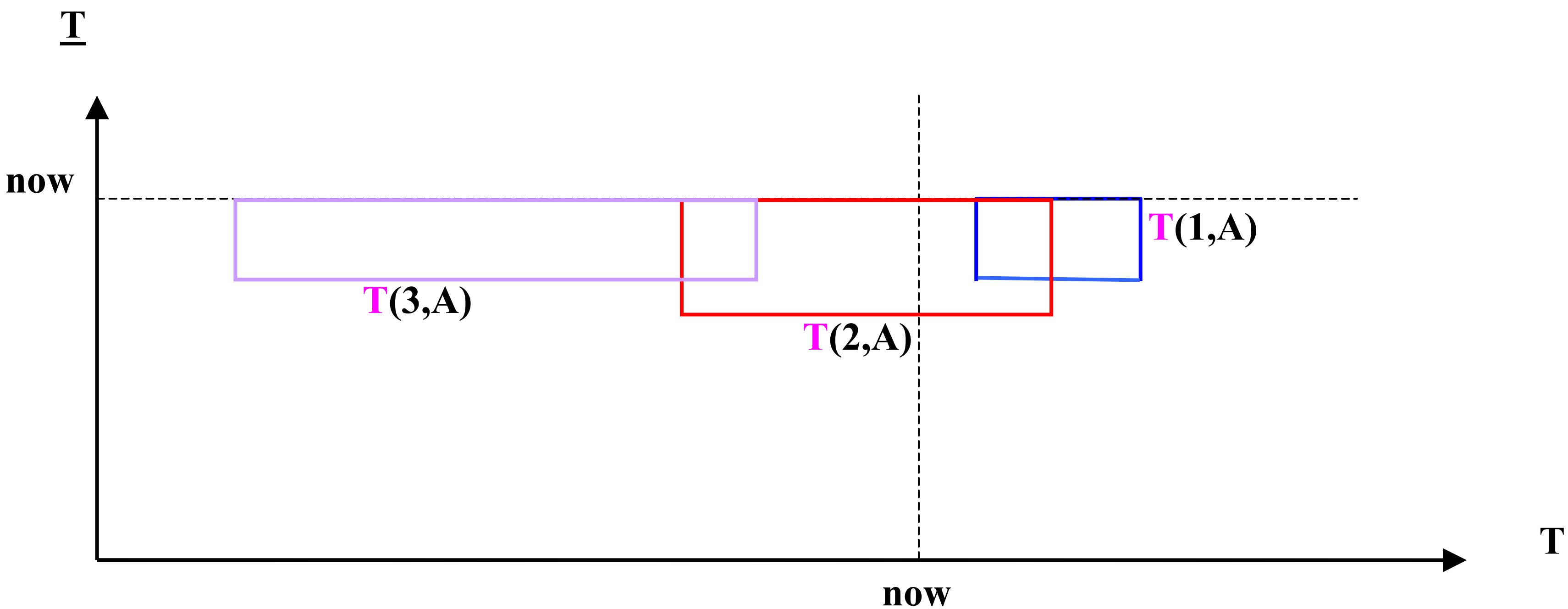
(6) $DB, \theta, t, \underline{t} \models \underline{\mathbf{X}}^{-}\varphi$ **iff**

there exists $\underline{t}_1 \in \underline{T}$: $\underline{t}_1+1 = \underline{t}$ **and** $DB, \theta, t, \underline{t}_1 \models \varphi$

Example of queries (1)

Patients that ended treatment with medicine A.

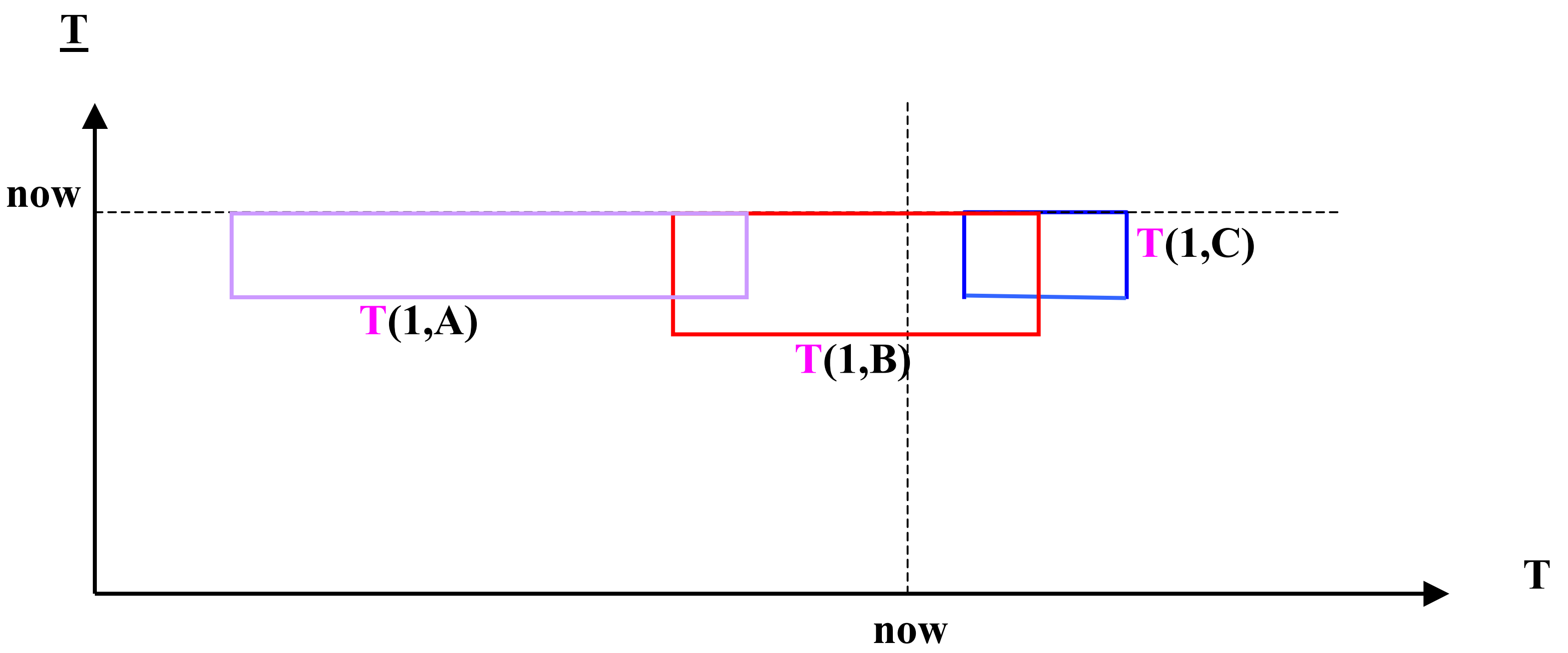
$$\varphi = \neg T(x,A) \wedge \mathbf{P}\, T(x,A) \wedge \neg \mathbf{F}\, T(x,A)$$
$$\wedge\ date(now) \wedge \underline{date}(now)$$



Example of queries (2)

The treatment of patient Kowalski

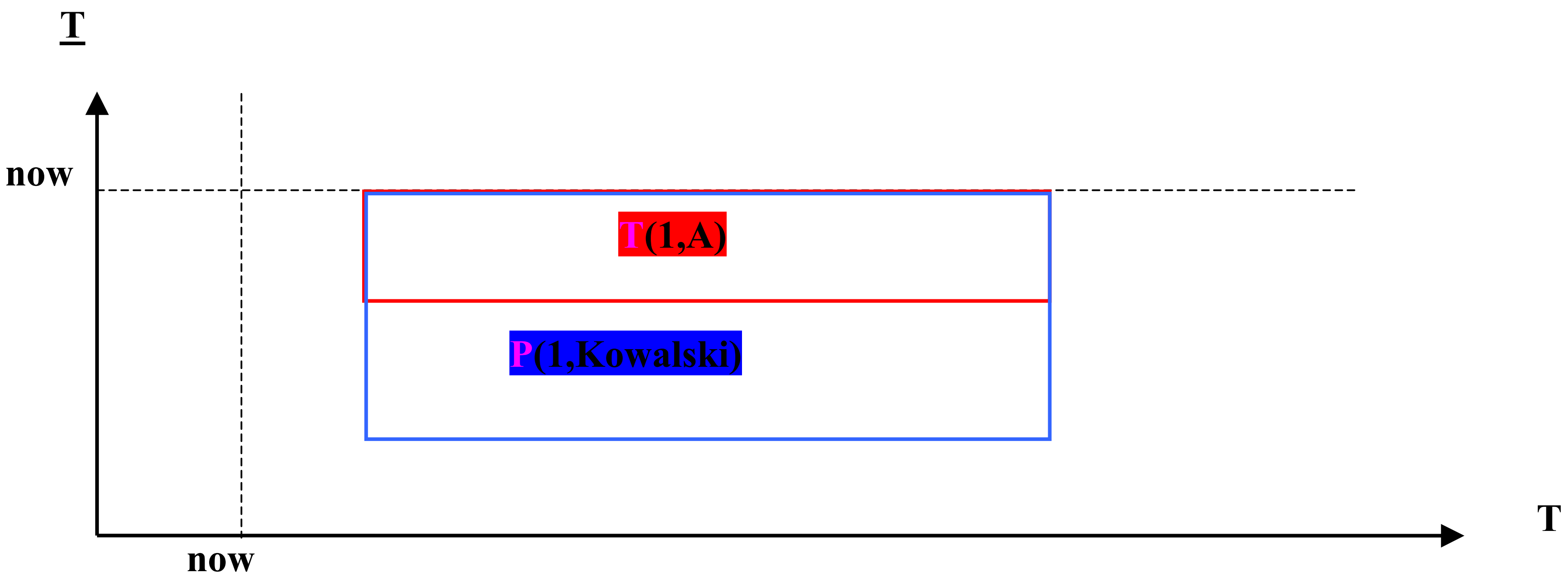
$$\varphi = [\mathbf{P}\,T(1,x) \vee T(1,x) \vee \mathbf{F}T(1,x)] \wedge \text{date}(\text{now}) \wedge \underline{\text{date}}(\text{now})$$



Example of queries (3)

All the will-be patients who will be treated with A for whole stay in the hospital

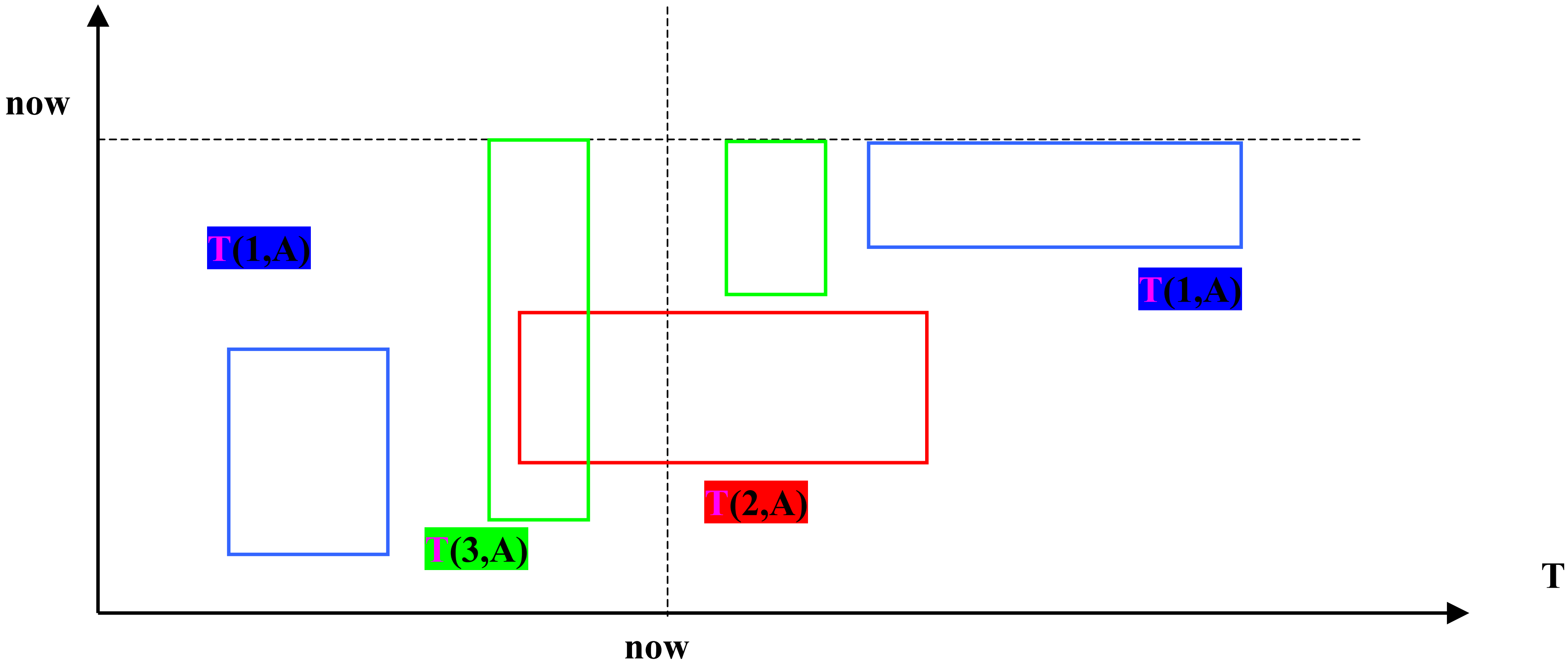
$$\varphi = \mathbf{F} \langle P(x,y) \wedge \mathbf{X}^{-} \neg P(x,y) \wedge T(x,A) \wedge \{[P(x,y) \wedge T(x,A)] \mathbf{U} [P(x,y) \wedge \mathbf{X} \neg P(x,y) \wedge T(x,A)]\} \rangle \wedge date(now) \wedge \underline{date}(now)$$



Example of queries (4)

Identifiers of patients whose records of treatment with A have been deleted

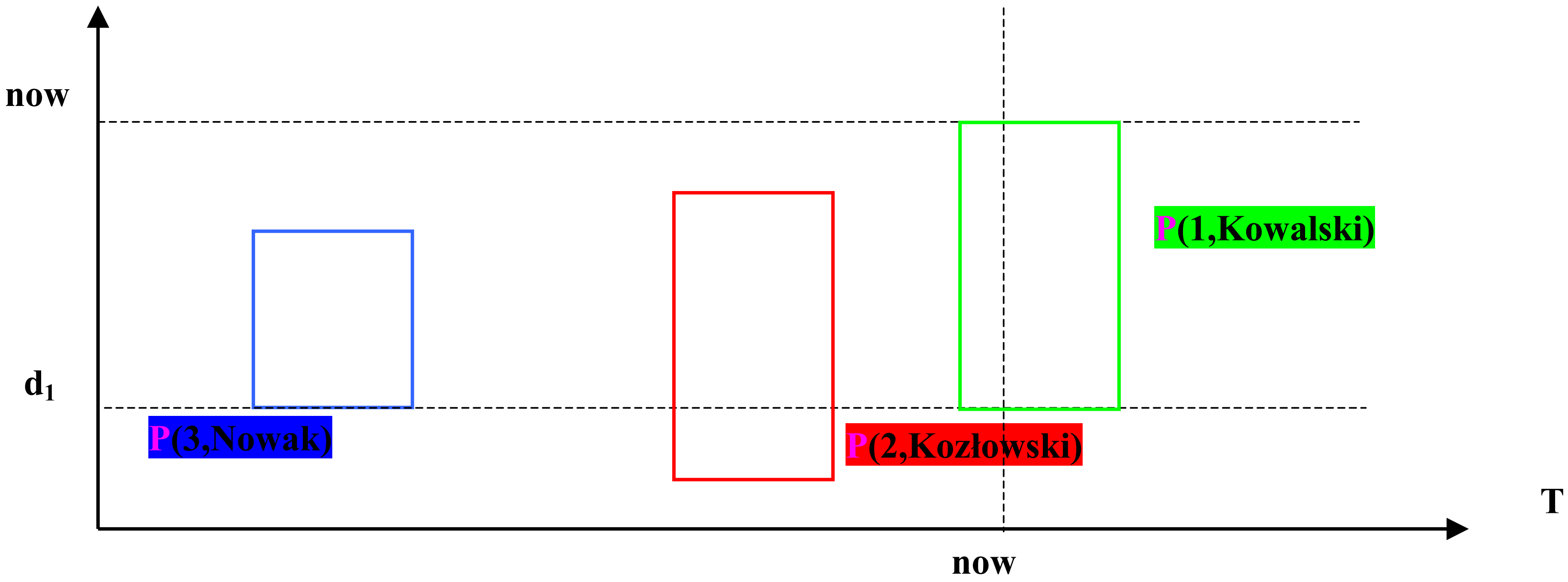
$$\varphi = \underline{\mathbf{P}}_{\underline{T}} \, [T(x,A) \wedge \underline{\mathbf{X}} \neg T(x,A)] \wedge \underline{date}(now)$$



Example of queries (5)

Patients whose records of stay in the hospital were inserted on the day d_1 but have been eventually deleted from the database

$$\varphi \underset{\mathbf{\underline{T}}}{=} P(x,y) \wedge \underline{\mathbf{X}}^{-} \neg P(x,y) \wedge \underline{\mathbf{F}} \neg P(x,y) \wedge \underline{\text{date}}(d_1)$$



Example of queries (6)

Patients whose records of stay in the hospital were inserted on the day d_1 and are held in the database until now

$$\varphi = \{P(x,y)\ \underline{\mathbf{S}}\ [P(x,y) \wedge \underline{\mathbf{X}}^{-} \neg P(x,y) \wedge \underline{\text{date}}(d_1)]\}$$
$$\wedge\ P(x,y) \wedge \underline{\text{date}}(\text{now})$$